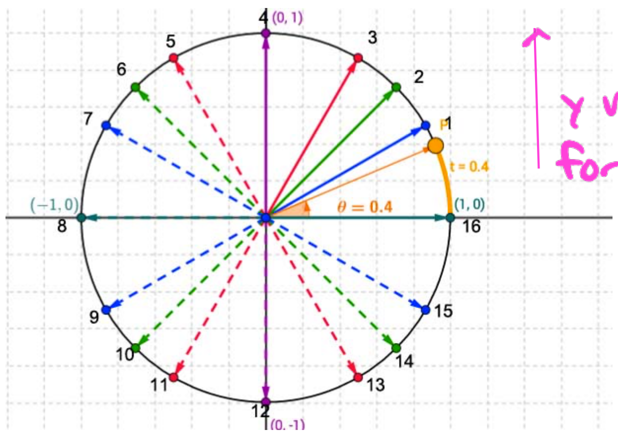
100 POINTS **NAME:** ____________________

Math 8 Test 1 (Take Home)
Fall 2024

- **Show all work on this paper.**
- **No scratch paper or notes.**
- **No graphing calculator.**
- **No credit will be given for solutions if work is not shown.**

(1) Same figure as on homework, see previous handout for colors.

The "blue angles" all have a reference angle of 30 degrees or $\pi/6$ radians.
The "green angles" all have a reference angle of 45 degrees or $\pi/4$ radians.
The "red angles" all have a reference angle of 60 degrees or $\pi/3$ radians.
(ignore the orange here) (12 points)



y values increasing for locations 1, 2, 3

Write the corresponding number for each of the following angles:

210°	9	$7\pi/6$	9	−120°	11
$5\pi/4$	10	315°	14	$\pi/3$	3
$11\pi/6$	15	$-3\pi/2$	4	$5\pi/3$	13
405°	2	$3\pi/4$	6	6π	16

405
360
45

If point $P\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$, $Q\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$, $R\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ are the coordinates of the points at location 1,2, and 3, which point goes with which location? 3 points

1) Q
2) P
3) R

y values increasing for locations 1, 2, 3

(2) Use matrix methods (Gaussian elimination or Gauss Jordan) to solve: (10 points)

$$\begin{cases} 3x+2y-5z=1 \\ 2x-3y-8z=1 \\ x+5y+2z=1 \end{cases}$$

You must obtain row echelon form or reduced row echelon form. Be sure to label operations performed at each step.

$$\left[\begin{array}{ccc|c} 3 & 2 & -5 & 1 \\ 2 & -3 & -8 & 1 \\ 1 & 5 & 2 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} 1 & 5 & 2 & 1 \\ 2 & -3 & -8 & 1 \\ 3 & 2 & -5 & 1 \end{bmatrix} \xrightarrow[-3R_1+R_3\to R_3]{-2R_1+R_2\to R_2}$$

$$\begin{bmatrix} 1 & 5 & 2 & 1 \\ 0 & -13 & -12 & -1 \\ 0 & -13 & -11 & -2 \end{bmatrix} \xrightarrow{-R_2+R_3\to R_3} \begin{bmatrix} 1 & 5 & 2 & 1 \\ 0 & -13 & -12 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow{-\frac{1}{13}R_2\to R_2}$$

$$\begin{bmatrix} 1 & 5 & 2 & 1 \\ 0 & 1 & 12/13 & 1/13 \\ 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow[-2R_3+R_1\to R_1]{-\frac{12}{13}R_3+R_2\to R_2} \begin{bmatrix} 1 & 5 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Row Ech. Form

$$\xrightarrow[-5R_2+R_1\to R_1]{} \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\Longrightarrow \boxed{(-2, 1, -1)}$$

checks in system

(3) Use Cramer's Rule to solve the following system. $\begin{cases} 3x-3y=5 \\ -x+5y=7 \end{cases}$ (8 points)

(No credit given for a different method)

$$D = \begin{vmatrix} 3 & -3 \\ -1 & 5 \end{vmatrix} = 15-3=12$$

$$D_x = \begin{vmatrix} 5 & -3 \\ 7 & 5 \end{vmatrix} = 25 - {}^-21 = 46$$

$$D_y = \begin{vmatrix} 3 & 5 \\ -1 & 7 \end{vmatrix} = 21+5=26$$

$$x = \frac{D_x}{D} = \frac{46}{12} = \frac{23}{6}$$

$$y = \frac{D_y}{D} = \frac{26}{12} = \frac{13}{6}$$

$$\boxed{\left(\frac{23}{6}, \frac{13}{6}\right)}$$

Check in system

$$3 \cdot \frac{23}{6} - 3\left(\frac{13}{6}\right) = \frac{23}{2} - \frac{13}{2} = 5 \checkmark$$

$$-\left(\frac{23}{6}\right) + 5\left(\frac{13}{6}\right) = \frac{-23}{6} + \frac{65}{6} = \frac{42}{6} = 7 \checkmark$$

(4) Given the following matrices: (a-d, 2 points each; e,f 4 points each)

$$A = \begin{bmatrix} 9 & -2 \\ -1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 1 & -2 \\ -3 & 0 & 5 \\ 8 & 1 & -6 \end{bmatrix} \quad C = \begin{bmatrix} 7 & 0 \\ 5 & -2 \end{bmatrix} \quad D = \begin{bmatrix} 6 & 2 & -4 \\ 1 & 0 & 7 \end{bmatrix}$$

Find the following, if possible. (If not possible, say so.)

(a) A + C

$$\boxed{\begin{bmatrix} 16 & -2 \\ 4 & 1 \end{bmatrix}}$$

(b) AC

$$\boxed{\begin{bmatrix} 53 & 4 \\ 8 & -6 \end{bmatrix}}$$

(c) BC

$(3\times3)(2\times2)$

not same -

$\boxed{\text{cannot multpy}}$

(d) det(C)

$$\begin{vmatrix} 7 & 0 \\ 5 & -2 \end{vmatrix} = -14 - 0 = \boxed{-14}$$

(e) AD

$$\boxed{\begin{bmatrix} 52 & 18 & -50 \\ -3 & -2 & 25 \end{bmatrix}}$$

(f) det (B)

$$= \begin{vmatrix} 4 & 1 & -2 \\ -3 & 0 & 5 \\ 8 & 1 & -6 \end{vmatrix} = \boxed{8}$$

$$-(-3)\begin{vmatrix} 1 & -2 \\ 1 & -6 \end{vmatrix} - 5\begin{vmatrix} 4 & 1 \\ 8 & 1 \end{vmatrix}$$

$$3(-4) - 5(-4)$$

$$= -12 + 20 = 8$$

check by using a different row

$$4\begin{vmatrix} 0 & 5 \\ 1 & -6 \end{vmatrix} - 1\begin{vmatrix} -3 & 5 \\ 8 & -6 \end{vmatrix} - 2\begin{vmatrix} -3 & 0 \\ 8 & 1 \end{vmatrix}$$

$$= 4(-5) - (-22) - 2(-3)$$

$$= -20 + 22 + 6 = 8$$

(5) Given $A = \begin{bmatrix} 1 & -2 & -4 \\ 2 & -3 & -6 \\ -3 & 6 & 15 \end{bmatrix}$

(a) Find A^{-1} (10 points)

(b) Use A^{-1} to solve the system $\begin{cases} x-2y-4z=2 \\ 2x-3y-6z=0 \\ -3x+6y+15z=1 \end{cases}$ (3 points)

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & -2 & -4 & 1 & 0 & 0 \\ 2 & -3 & -6 & 0 & 1 & 0 \\ -3 & 6 & 15 & 0 & 0 & 1 \end{array}\right] \xrightarrow[3R_1+R_3 \to R_3]{-2R_1+R_2 \to R_2} \left[\begin{array}{ccc|ccc} 1 & -2 & -4 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & 3 & 3 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{\frac{1}{3}R_3 \to R_3} \left[\begin{array}{ccc|ccc} 1 & -2 & -4 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1/3 \end{array}\right] \xrightarrow{-R_3+R_2 \to R_2} \left[\begin{array}{ccc|ccc} 1 & -2 & -4 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & -2/3 \\ 0 & 0 & 1 & 1 & 0 & 1/3 \end{array}\right]$$

$$\xrightarrow{4R_3+R_1 \to R_1} \left[\begin{array}{ccc|ccc} 1 & -2 & 0 & 5 & 0 & 4/3 \\ 0 & 1 & 0 & -4 & 1 & -2/3 \\ 0 & 0 & 1 & 1 & 0 & 1/3 \end{array}\right]$$

$$\xrightarrow{2R_2+R_1 \to R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 0 & -4 & 1 & -2/3 \\ 0 & 0 & 1 & 1 & 0 & 1/3 \end{array}\right]$$

$$\boxed{A^{-1} = \begin{bmatrix} -3 & 2 & 0 \\ -4 & 1 & -2/3 \\ 1 & 0 & 1/3 \end{bmatrix}}$$

checked

$$A^{-1}A = \begin{bmatrix} -3 & 2 & 0 \\ -4 & 1 & -2/3 \\ 1 & 0 & 1/3 \end{bmatrix} \begin{bmatrix} 1 & -2 & -4 \\ 2 & -3 & -6 \\ -3 & 6 & 15 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) Solution is $A^{-1}B = \begin{bmatrix} -3 & 2 & 0 \\ -4 & 1 & -2/3 \\ 1 & 0 & 1/3 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ -26/3 \\ 7/3 \end{bmatrix}$

$$\boxed{\left(-6, -\frac{26}{3}, \frac{7}{3}\right)}$$

can check in system

(6) (a) Convert from DMS (degree, minute seconds)to decimal degrees, show work. $23^\circ 15' 42''$
(8 points)

$15' \cdot \frac{1^\circ}{30'} = \frac{1}{2}^\circ = .5^\circ$ $\quad 42'' \cdot \frac{1'}{60''} \cdot \frac{1^\circ}{60'} = \frac{42}{3600}^\circ = .011\overline{6}$ $\quad \boxed{23.26}$

(b) Convert from decimal degrees to DMS , show work. 38.4°

$38^\circ \quad .4^\circ \cdot \frac{60'}{^\circ} = 24' \Rightarrow \boxed{38^\circ 24'}$

(c) Convert from radians to degrees: $\frac{3\pi}{10}$

$\frac{3\pi}{10} \cdot \frac{180}{\pi} = \boxed{54^\circ}$

(d) Convert from degrees to radians, exactly (no calculator): 26°

$26^\circ \cdot \frac{\pi}{180} = \frac{26\pi}{180} = \boxed{\frac{13\pi}{90}}$

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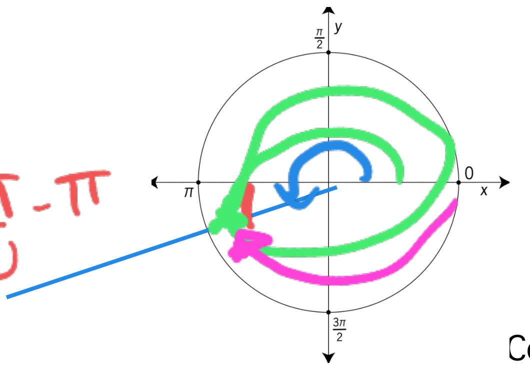
(7) Graph the angle $\theta = 9\pi/8$ in standard position. Give two coterminal angles, one of which is positive and the other negative. Find the reference angle. (8 points)

$\frac{9\pi}{8} - \pi$



Coterminal positve $\frac{25\pi}{8}$ Coterminal negative $-\frac{7\pi}{8}$ Ref angle $\frac{\pi}{8}$

$\frac{9\pi}{8} + 2\pi$

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(8) (For each of the following acute angles, find 4 angles, one in each quadrant, having the given angle as a reference angle. Answer in the units given, exactly. (12 points)

	Q1	Q2	Q3	Q4
17°	17°	163°	197°	343°
$2\pi/7$	$2\pi/7$	$5\pi/7$	$9\pi/7$	$12\pi/7$
1	1	$\pi - 1$	$\pi + 1$	$2\pi - 1$

general radians $\quad \theta \quad \theta \quad \pi - \theta \quad \pi + \theta \quad 2\pi - \theta$

(9) Solve using any of the methods discussed in class. (10 points)

$$\begin{cases} x+y-10z=-4 \\ -3x-5y+36z=10 \\ -x \quad +7z=5 \end{cases}$$

$$\left[\begin{array}{cccc} 1 & 1 & -10 & -4 \\ -3 & -5 & 36 & 10 \\ -1 & 0 & 7 & 5 \end{array}\right] \xrightarrow[R_1+R_3\to R_3]{3R_1+R_2\to R_2} \left[\begin{array}{cccc} 1 & 1 & -10 & -4 \\ 0 & -2 & 6 & -2 \\ 0 & 1 & -3 & 1 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{2}R_2\to R_2} \left[\begin{array}{cccc} 1 & 1 & -10 & -4 \\ 0 & 1 & -3 & 1 \\ 0 & 1 & -3 & 1 \end{array}\right] \xrightarrow{-R_2+R_3\to R_3}$$

$$\left[\begin{array}{cccc} 1 & 1 & -10 & -4 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \leftarrow \text{Infinitely many solutions}$$

$$\begin{cases} x+y-10z=-4 \\ y-3z=1 \end{cases} \Rightarrow$$

$x = -y+10z-4 = 0$

$y = 3z+1$ subst

$x = -(3z+1)+10z-4$

$x = 7t-5$

$$(7t-5,\ 3t+1,\ t)$$